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Reporting Period: April 1 to June 30, 1984

*Expenditures for period*

Salaries  
Benefits  
Travel  
Computer time  
Telephone  
Xeroxing  
Mail  
Supplies & expenses  
Dept. recharges

TOTAL

**Presentations and Publications by Principal Investigator and Graduate Students**

Dozier, J., 1984, Reflectance measurements with Landsat Thematic Mapper over rugged terrain, *Tenth International Symposium, Machine Processing of Remotely Sensed Data*, 230-234.

Dozier, J., 1984, Snow reflectance from Landsat-4 Thematic Mapper, *IEEE Transactions on Geoscience and Remote Sensing*, GE-22, 323-328.

Frew, J., 1984, Registering Thematic Mapper imagery to digital elevation models, *Tenth International Symposium, Machine Processing of Remotely Sensed Data*, 432-435.  
(Copies attached.)

**Development of Twostream Model (paper in preparation)**

**INTRODUCTION**

Two-stream methods provide rapid approximate calculations of radiative transfer in scattering and absorbing media. Although they provide information on fluxes only, and not on intensities, their speed makes them attractive alternatives to more precise methods.

*Meador and Weaver* [1980] provide a comprehensive, unified review of two-stream methods for a homogeneous layer, and solve the equations for reflectance and transmittance for a homogeneous layer over a non-reflecting surface. In this paper, we show how any of the basic kernels they describe for a single layer can be extended to a vertically inhomogeneous medium over a

surface whose reflectance properties vary with illumination angle, as long as the medium can be subdivided into homogeneous layers. The method never produces nonphysical results (such as negative albedo), and it is computationally stable, without any restrictions on the optical properties of the layer.

The outline of the procedure is:

1. For the bottom layer, over a surface whose reflectance properties are known, solve the two-stream equations to calculate reflectance and transmittance for direct and diffuse irradiance.
2. The bottom layer can now be considered the new "surface," whose reflectance properties are known, so successively calculate reflectances and transmittances for each layer to the top of the medium.
3. Now start at the top with known incident flux, and successively calculate upward and downward fluxes for each layer.

## TWO-STREAM APPROXIMATION FOR A HOMOGENEOUS LAYER

*Meador and Weaver* [1980] review the derivation for two-stream methods, so the treatment here is brief. They show that all of the common variants, including the Eddington and delta-Eddington methods, are different cases of the same differential equations.

The optical properties of a layer are described by:

- $\omega$  Single-scattering albedo, the portion of the radiation incident on a scattering element that is reflected, instead of absorbed. It is defined by the ratio of scattering to extinction efficiency.
- $g$  Mean value of the cosine of the scattering angle, measured from the forward direction.  $g = +1$  corresponds to completely forward scattering,  $-1$  to completely backward scattering, and  $0$  to isotropic scattering.
- $\tau_0$  Optical thickness of the layer.

Direct flux  $E_0$ , measured perpendicular to the beam, is incident at angle  $\arccos \mu_0$ . The diffuse upward and downward fluxes  $F^+$  and  $F^-$  at optical depth  $\tau$  within the layer are described by the following pair of equations for any two-stream approximation:

$$\frac{dF^+}{d\tau} = \gamma_1 F^+ - \gamma_2 F^- - \omega E_0 \gamma_3 e^{-\tau/\mu_0} \quad (1)$$

$$\frac{dF^-}{d\tau} = \gamma_2 F^+ - \gamma_1 F^- + \omega E_0 \gamma_4 e^{-\tau/\mu_0} \quad (2)$$

The layer is homogeneous if the  $\gamma$ 's are independent of  $\tau$ . The choice of the  $\gamma$ 's is determined by the particular approximation to the scattering phase function and the radiation intensity distribution. One constraint is that  $\gamma_3 + \gamma_4 = 1$ , because of energy conservation. *Meador and Weaver* [1980] give expressions for the  $\gamma$ 's for 7 different two-stream approximations.

## GENERAL SOLUTIONS

General solutions to (1, 2) differ for non-conservative ( $\omega < 1$ ) and conservative ( $\omega = 1$ ) scattering [Wiscombe, 1977].

### 1. Non-Conservative Scattering

A general solution for the case  $\omega < 1$ , is

$$F^+ = E_0 \left[ \frac{C_1 (\xi + \gamma_1)}{\gamma_2} e^{\xi\tau} - \frac{C_2 (\xi - \gamma_1)}{\gamma_2} e^{-\xi\tau} - Q e^{-\tau/\mu_0} \right] \quad (3)$$

$$F^- = E_0 \left[ C_1 e^{\xi\tau} + C_2 e^{-\xi\tau} - P e^{-\tau/\mu_0} \right] \quad (4)$$

where

$$\xi^2 = \gamma_1^2 - \gamma_2^2$$

$$P = k \mu_0 (\alpha_1 \mu_0 + \gamma_4)$$

$$Q = k \mu_0 (\alpha_2 \mu_0 - \gamma_3)$$

$$\alpha_1 = \gamma_1 \gamma_4 + \gamma_2 \gamma_3$$

$$\alpha_2 = \gamma_1 \gamma_3 + \gamma_2 \gamma_4$$

$$k = \frac{\omega}{1 - \xi^2 \mu_0^2}$$

$C_1$  and  $C_2$  are arbitrary constants. Given top and bottom boundary conditions, their expressions can be derived, and the general solution can be used to find reflectance and transmittance of a layer.

## 2. Conservative Scattering

When  $\omega = 1$ , a general solution is

$$F^+ = E_0 \left[ C_1 + C_2 \tau - \mu_0 (\gamma_1 \mu_0 - \gamma_3) e^{-\tau/\mu_0} \right] \quad (5)$$

$$F^- = E_0 \left[ C_1 + C_2 \left( \tau - \frac{1}{\gamma_1} \right) - \mu_0 (\gamma_1 \mu_0 + \gamma_4) e^{-\tau/\mu_0} \right] \quad (6)$$

where  $C_1$  and  $C_2$  again are arbitrary constants.

## BOUNDARY CONDITIONS

The coefficients  $C_1, C_2$  depend on the boundary conditions. The usual top boundary condition is that there is no diffuse irradiance at the top of the layer.

$$F^-(0) = 0 \quad (7)$$

In atmospheric radiation models the usual bottom boundary condition is simple. *Meador and Weaver* [1980] for example use a flat, unobstructed, zero-albedo surface; *Wiscombe and Warren* [1980] use a flat, unobstructed, Lambertian surface. To apply an atmospheric model over the land however, we must allow for the surface reflectance properties to vary with incidence angle, and for irregular terrain such that the illumination angle at the surface is not necessarily the same as at the top of the atmosphere. Moreover we must recognize that a significant portion of the incident diffuse irradiance comes from reflection from adjacent terrain.

Define the following quantities:

- $\mu_0$  cosine of illumination angle at the local surface;
- $\rho'_d$  bihemispherical reflectance of the surface;
- $\rho'_s(\mu_0)$  directional-hemispherical reflectance of the surface at incidence angle  $\cos^{-1} \mu_0$ ;
- $V_d$  terrain "view factor" for incident diffuse irradiance;
- $V_s$  terrain view factor for incident direct irradiance.

Then the lower boundary condition is

$$F^+(\tau_0) = E_0 e^{-\tau_0/\mu_0} [\rho'_s \mu_0 + \rho'_d V_s] + F^-(\tau_0) \{ \rho'_d [1 - (1 - \rho'_d) V_d] \} \quad (8)$$

The terms bihemispherical and directional-hemispherical reflectance are defined by *Nicodemus et al.* [1977] to describe the hemispherically integrated reflectance to diffuse and direct irradiance. The view factor  $V_d$  for diffuse irradiance accounts for the portion of the hemisphere "seen" by a point, weighted by the angles between the point and the surrounding terrain in the range of azimuth directions.  $S$  is the slope angle,  $E$  is the exposure, the direction that

the slope faces, and  $H_p$  is the angle to the horizon in direction  $\varphi$ .

$$V_d = \int_{-\pi}^{\pi} [\cos S \cos H_p + \sin S \cos H_p \cos(\varphi - E)]^2 d\varphi \quad (9)$$

The view factor  $V_s$  for direct irradiance is similar to  $V_d$ , except that the integrand for each direction  $\varphi$  is weighted by the cosine of the illumination angle on the slope to the horizon, i.e. by  $[\mu_0 \cos H_p - (1 - \mu_0^2)^{1/2} \sin H_p \cos(\varphi_0 - \varphi)]$ , where this term is set to zero if it is negative.  $\varphi_0$  is the azimuth of illumination.

Simplified bottom boundary conditions can be implemented by modifying some of the above terms. If the surface is Lambertian, for example, set  $\rho'_s = \rho'_d$ . If the surface is flat, set  $\mu_0 = \mu_s$ . If the surface is unobstructed by surrounding terrain, set  $V_d = V_s = 0$ . Even in rugged terrain we can often assume  $V_s \approx \mu_0 V_d$ .

### REFLECTANCE OF A HOMOGENEOUS LAYER

Directional-hemispherical reflectance of the layer (also called direct albedo) is the upward radiance emerging from the layer, integrated over the hemisphere, divided by the direct incident irradiance. Bi-hemispherical reflectance (also called diffuse albedo) is the upward radiance divided by the downward diffuse irradiance, and can be derived by integrating directional-hemispherical reflectance over the hemisphere.

$$\rho_s(\mu_0) = \frac{F^+(0)}{\mu_0 E_0} \quad (10)$$

$$\rho_d = \frac{F^+(0)}{F^-(0)} = 2 \int_0^1 \mu_0 \rho_s d\mu_0 \quad (11)$$

Direct albedo is calculated by finding the values for  $C_1$  and  $C_2$  in the general solution, given the boundary conditions, and then substituting these values in (10).

Integration of  $\rho_s$  over  $\mu_0$  to obtain  $\rho_d$  is done numerically, although for particular approximations to the phase function analytic integration is possible. For example, *Wiscombe and Warren* [1980] give an analytic integration for the delta-Eddington approximation.

#### 1. Non-Conservative Scattering

For  $\omega < 1$ , direct reflectance is

$$\rho_s(\mu_0) = \frac{2\xi\psi e^{-\tau_0/\mu_0}}{D_h} + \frac{k(\alpha_1\mu_0 + \gamma_4)}{D_h} [H^- e^{-\tau_0} + H^+ e^{\tau_0}] - k(\alpha_2\mu_0 - \gamma_3) \quad (12)$$

where

$$D_h = G^- e^{-\tau_0} + G^+ e^{\tau_0}$$

$$G^\pm = \xi \mp \delta \gamma_2 \pm \gamma_1$$

$$H^\pm = \delta(\xi \mp \gamma_1) \pm \gamma_2$$

$$\delta = \rho'_d [1 - (1 - \rho'_d) V_d]$$

$$\psi = \frac{\eta}{\mu_0} - k[\delta(\alpha_1\mu_0 + \gamma_4) - \alpha_2\mu_0 + \gamma_3]$$

$$\eta = \rho'_s \mu_0 + \rho'_d V_s$$

When the layer is semi-infinite (i.e. when  $\tau_0 \rightarrow \infty$ ) reflectance is:

$$\rho_s^{(\tau_0 \rightarrow \infty)}(\mu_0) = \frac{\alpha(\gamma_1 - \xi\gamma_4)}{\gamma_3(1 + \xi\mu_0)} \quad (13)$$

## 2. Conservative Scattering

In the conservative scattering case, when  $\omega = 1$ , direct reflectance is

$$\rho_s^{(\omega=1)}(\mu_0) = 1 - \frac{1}{D_0} \times \left\{ (1-\delta)(\gamma_1\mu_0 + \gamma_4) - e^{-\tau_0/\mu_0} \left[ (1-\delta)(\gamma_1\mu_0 + \gamma_4) + \frac{\eta}{\mu_0} - 1 \right] \right\} \quad (14)$$

where

$$D_0 = 1 + \tau_0\gamma_1(1-\delta)$$

## TRANSMITTANCE OF A HOMOGENEOUS LAYER

We distinguish three kinds of transmittances:

Directional transmittance  $T_s$  is the proportion of the direct irradiance incident on the top of the layer that is transmitted without scattering or absorption.

$$T_s(\mu_0) = e^{-\tau_0/\mu_0} \quad (15)$$

Directional-hemispherical transmittance  $T_{sd}$  is the proportion of the direct irradiance incident on the top of the layer that emerges as diffuse irradiance at the bottom of the layer, after scattering within it.

$$T_{sd}(\mu_0) = \frac{F^-(\tau_0)}{\mu_0 E_0} \quad (16)$$

Bihemispherical transmittance  $T_d$  is the proportion of the incident diffuse irradiance that emerges from the bottom of the layer.

$$T_d = 2 \int_0^1 \mu_0 (T_s + T_{sd}) d\mu_0 \quad (17)$$

## 1. Non-Conservative Scattering

When  $\omega < 1$ , directional-hemispherical transmittance is

$$T_{sd}(\mu_0) = \frac{2k\xi(\alpha_1\mu_0 + \gamma_4)}{D_n} + e^{-\tau_0/\mu_0} \left[ \frac{\psi\gamma_2(e^{\tau_0} - e^{-\tau_0})}{D_n} - k(c_1\mu_0 + \gamma_4) \right] \quad (18)$$

## 2. Conservative Scattering

When  $\omega = 1$  directional-hemispherical transmittance is

$$T_{sd}^{(\omega=1)}(\mu_0) = 1 - [\rho_s^{(\omega=1)}(\mu_0) + e^{-\tau_0/\mu_0}] = \frac{1}{D_0} \times \left\{ (1 - e^{-\tau_0/\mu_0})(\gamma_1\mu_0 + \gamma_4) + \gamma_1\tau_0 \left[ \frac{\eta}{\mu_0} - 1 \right] e^{-\tau_0/\mu_0} \right\} \quad (19)$$

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